Week 3 Quiz

1.) Use proof by contradiction to show that the set of prime numbers is infinite.

**Proof(by contradiction):**

**Suppose not. That is, suppose that the set of prime numbers is finite. This statement implies that there is a largest prime number. Let the list of all prime numbers be represented by where is the largest prime number.  
Multiplying all the primes together can be represented by   
Let   
 cannot be evenly divided by any of these prime numbers. For example,  
   
Thus, does not evenly divide, which implies is a prime number bigger than . This is true for and so on as well. Therefore, because is not divisible by any of through , and is bigger than we have a contradiction [what we needed to show.]**

2.) Suppose a ∈  Z. If a3is even, then a is even. Use proof by contradiction.

**Let P = is even”, and Q = “ is even”. Assume for the sake of contradiction or meaning that is even and is not even, which makes odd.  
By definition of even, for some integer , and by definition of odd, for some integer . is an integer because it is the product and sum of integers.  
By substitution,**

**By definition in Example 4.2.3, an odd integer times another odd integer is odd. The odd result multiplied by another odd integer is still odd. Thus,   
 is odd, which contradicts our assumption that is even. [Hence the supposition is false and the theorem is true.]**

3.) Let A ={x  ∈  Z | x =10a+7  for some integer a }   
and  B={y  ∈  Z | y = 10b − 3 for some integer b}.

Prove or disprove that  B  ⊆ A

**Suppose is a particular but arbitrarily chosen element of B. [We must show that . By definition of A, this means we must show that .] By definition of B, there is an integer such that . [Given that , can also be expressed as ? I.e., is there an integer, say such that Solve for to obtain Check to see if this works.] Let . [First check that is an integer.] Then is an integer because it is the difference of integers. [Then check that ]   
Also   
Thus, by definition of A, is an element of A [which is what was to be shown.]**

4.) False

5.) False

6.) False

7.) True

8.)

9.) Let,  A={{1}} ,  B={1, {2}}  and C = {{1,2}}. Find  (A X B) X C .